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DETERMINATION OF THE RESISTANCE COEFFICIENT OF THE ROUND PIPELINE

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ОПРЕДЕЛЕНИЕ КОЭФФИЦИЕНТА СОПРОТИВЛЕНИЯ КРУГЛОГО ТРУБОПРОВОДА

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The value of the resistance coefficient of the round pipeline with the arbitrary Reynolds number and degrees of roughness is known only from the experiment. With a consolidated solution, the author offers to obtain the solution of the Navier-Stokes equations and to define the influence of roughness on its solution on the basis of qualitative considerations. With an accuracy of 10%, the author made the classical Nikuradze's graphs of dependence of resistance coefficient of a circular pipeline on the arbitrary Reynolds number with different degrees of roughness.

Keywords: resistance; coefficient; turbulent regime; Nikuradze's graphs.

Значение коэффициента сопротивления круглого трубопровода при произвольном числе Рейнольдса и степени шероховатости известно только из эксперимента. Предлагается с помощью комплексного решения получить решение уравZнения Навье – Стокса и на основе качественных соображений определить влияние шероховатости на решение уравнения Навье – Стокса. При этом с точностью 10% удалось построить классические графики Никурадзе зависимости коэффициента сопротивления круглого трубопровода в зависимости от произвольного числа Рейнольдса и степени шероховатости.

Ключевые слова: коэффициент сопротивления; турбулентный режим; графики Никурадзе

1. Introduction.

The problem of describing a fluid motion in a turbulent regime is not solved yet. This causes difficulties in calculating oil and gas pipelines. Moreover, there are no theoretical methods of describing body movements in a turbulent environment. It includes a description of the motion of an aircraft or a submarine, as well as a surface ship in the turbulent regime. Without the use of modeling a moving body in wind tunnels or water basins, this is impossible to design a body moving in a viscous medium. There are approximate formulas of approximation of the resistance coefficient of a pipeline in some areas of the Reynolds number (Monin & Yaglom, 1965; Schlichting, 1974). Furthermore, classical experimental graphs of Nikuradze for determination of the resistance coefficient of a circular pipeline as a function of the Reynolds number and degrees of roughness are also very well-known. Scientists use an approximation of the convective part that reduces the problem of solving the Navier-Stokes equations to the linear form with an effective turbulent viscosity. However, such a transformation distorts the solution of the Navier-Stokes equations, and the coefficient of eddy viscosity may have any value up to the negative one in order to coincide with the experiment. The Galerkin method is also applied, because it brings the solution of the problem of hydrodynamics to a system of nonlinear ordinary differential equations. However, this system of nonlinear equations has complex equilibrium positions in the turbulent regime, i. e. it may be considered comprehensive solution. Indeed, the system of equations in hydrodynamics in the turbulent regime in the real plane has no solution, because the solution of the equation tends to infinity (Yakubovsky, 2012). Additionally, its comprehensive solution is finite. The physical meaning of the complex solution and its oscillatory character of an imaginary part is shown in many studies, i.e. the solution of the problem of hydrodynamics in the turbulent regime can be found in the complex plane (Yakubovsky, 2014a; Yakubovsky, 2014b).

2. The algorithm for solving the problem of hydrodynamics in arbitrary geometry of the flow.

Navier-Stokes equations in Cartesian coordinates has the following form:

$$\frac{\partial V_i}{\partial t} + \sum_{k=1}^{3} V_k \frac{\partial V_i}{\partial x^k} = -\frac{\partial P}{\rho \partial x^i} + \nu \Delta V_i \,. \quad (1.1)$$

We solve a three-dimensional laminar problem without any convective term when a predetermined external force is g_l

$$\frac{\partial P}{\rho \partial x_i} = \nu \Delta V_i \,.$$

This problem is reduced to a dimensionless form by dividing ν^2 / d^3, then a dimensionless equation is obtained

$$\frac{\partial p}{\partial y_i} = \Delta R_i \,,$$

$$R_s = V_s d / \nu, p = \frac{P d^2}{\rho \nu^2}, y_s = s / d, h_s = g_s d^2 / \nu^2$$

The function is the solution of this problem

$$R_s(y_1, y_2, y_3) = -\int_V \frac{1}{4\pi |\mathbf{y} - \mathbf{z}|} \frac{\partial p}{\partial z_s} dz_1 dz_2 dz_3 \,.$$

Then we find the solution of a continuity equation according to external influence, when r_i is a response to this external influence

$$\frac{\partial R_i - r_i}{\partial x^i} = \int_V \frac{y_s - z_s}{4\pi |\mathbf{y} - \mathbf{z}|^2} \left(\frac{\partial p}{\partial z_s} - h_s\right) dz_1 dz_2 dz_3 = 0$$

From this, the equation to determine the pressure in the flow is obtained

$$\int_V \frac{y_s - z_s}{4\pi |\mathbf{y} - \mathbf{z}|^2} \frac{\partial p}{\partial z_s} dz_1 dz_2 dz_3 = \int_V \frac{y_s - z_s}{4\pi |\mathbf{y} - \mathbf{z}|^2} h_s dz_1 dz_2 dz_3 \,.$$

The pressure is obtained in the form $p=\sum_{n=0}^{N} a_n\varphi_n(z_1,z_2,z_3)$. One inserts it into the integrand, multiplies by $\varphi_m(y_1,y_2,y_3)$, integrates in space and gets a system of linear equations $b_m = A_{mn}a_n$, where we have the values of the coefficients

$$A_{mn}=\int_V\int_V \varphi_m(y_1,y_2,y_3)\frac{y_s-z_s}{4\pi|\mathbf{y}-\mathbf{z}|^2}\frac{\partial\varphi_n(z_1,z_2,z_3)}{\partial z_s}dz_1dz_2dz_3dy_1dy_2dy_3$$

$$b_m=\int_V\int_V \varphi_m(y_1,y_2,y_3)\frac{y_s-z_s}{4\pi|\mathbf{y}-\mathbf{z}|^2}h_s(z_1,z_2,z_3)dz_1dz_2dz_3dy_1dy_2dy_3,$$

where $h_l(y_1,y_2,y_3)$ is determined by external influences. One leads Navier-Stokes equations to dimensionless form and divides it by ν^2/d^3. As a result, we have a dimensionless equation

$$\frac{\partial\Re_l}{\partial\tau}+\sum_{k=1}^{3}\Re_k\frac{\partial\Re_l}{\partial y_k}=-\frac{\partial p}{\partial y_l}+\Delta\Re_l$$

$$\Re_l=\frac{V_l d}{\nu}, y_l=x_l/d, \tau=t\nu/d^2, p=\frac{Pd^2}{\rho\nu^2}, h_l=g_l\frac{d^2}{\nu^2}=\frac{\partial p}{\partial y_l}.$$

Navier-Stokes equations are multiplied by the cross section of the flow tube and recorded along the laminar solution

$$\Gamma_s=\int_{S_s}\Re_s ds_s/d^2$$

(see Kiloin, 1976).

In convective terms and the pressure gradient, one enters the derivative in the direction that corresponds to the direction of the flow lines of the laminar solution. Substituting into the equation, we have the solution in the following form:

$$\Gamma_s=\alpha_s(\tau)\int_{S_s} R_s[y_1(\alpha,\beta),y_2(\alpha,\beta),y_3(\alpha,\beta)]d\alpha d\beta/d^2, \quad (1.3)$$

where S_s is a section of the flow tube in the laminar regime.

These flow tubes are constructed in any current external force, which manifests itself in differential pressure. Further, it is necessary to take into account the roughness; under certain conditions, we get a complex turbulent solution that is connected with influence of a small quadratic convective term, which leads to a complex solution in high differential pressure. At the same time, the real solution, which is obtained by another sign of the module of standard deviation, should be excluded as an indicator that does not determine the fluctuating turbulent solution. It is worth noting that an imaginary part of the solution determines pulsations of the solutions. The boundary between the laminar real solution and the turbulent comprehensive solution appears in this case. When selecting another sign of the square root and taking into account the correlation function of the process $<u'_l\ u'_k>$, where u'_k is a speed deviation from the mean value, the turbulent viscosity becomes negative.

Introduction of roughness leads to dependence $\Gamma_s(s)$, because functions $y_l(\alpha,\beta,s), l=1,\ldots,3$ becomes dependent on S through dependence of the radius $a_0(s)$ on the degree of micro roughness. Next, we highlight da_0/ds that is related to roughness; we find its rms value. At the same time, one makes an averaged equation according to S. When substituted in the Laplace operator and in the derivative with respect to the direction, we must consider the Laplace operator first and only then integrate the flow tube. As a result, the convective term in the laminar regime equals zero. However, if we add $\Re_n$ in the form $\Re_s = \frac{\int_{S_s} \Re_s d\alpha d\beta}{S_s} = \frac{\Gamma_s d^2}{S_s}$, we obtain the following equation:

$$\frac{\partial \Gamma_s}{\partial \tau} + \frac{\Gamma_s d^2}{S_s} \frac{\partial \Gamma_s}{\partial s} = -\frac{\partial \int_{S_s} \frac{p d\alpha d\beta}{d^2}}{\partial s} + \Delta\Gamma_s, S_s = \int_{S_s} d\alpha d\beta, \Gamma_s = \int_{S_s} \Re_s d\alpha d\beta / d^2 .$$

In order to take into account the surface roughness of a pipeline and to receive the turbulent solution, we should consider the rms of the tangent of slope roughness. In this case, the convective term is small, but, meanwhile, it is different from zero and is proportional to the rms of the tangent of slope roughness $<| \frac{da_0}{ds} - < \frac{da_0}{ds} > |>$ in the presence of a part that is associated with an impermanent cross section of the pipeline $\frac{d < a_0 >}{ds}$. Thus, the complex turbulent solution remains the flow lines of a laminar solution. Moreover, these pulsations are determined by the imaginary part of velocity. The imaginary part of the solution, which is equal to a constant, means pulsations with an amplitude that are equal to imaginary parts of velocity.

If we substitute the solution (1.1.2) in Navier-Stokes equations, integrate it into the flow tubes, multiply by R_{cr}, where this value satisfies the condition $1/R_{cr} = <| \tan\alpha |>$, where $<| \tan\alpha |>$ is the rms of the tangent of slope roughness, we receive the equation

$$R_{cr} \frac{d\alpha_s(\tau)}{d\tau} = F_s \alpha_s^2 - 2R_{cr}\alpha_s G_s + H_s$$

$$F_s = \int_V \frac{\Gamma_s d^2}{S_s} [y_1(\alpha,\beta,s), y_2(\alpha,\beta,s), y_3(\alpha,\beta,s)] \times \cdot$$

$$\times \frac{\partial R_n [y_1(\alpha,\beta,s), y_2(\alpha,\beta,s), y_3(\alpha,\beta,s)]}{\partial s} d\alpha d\beta ds$$

$$G_s = -\int_V \Delta R_s [y_1(\alpha,\beta,s), y_2(\alpha,\beta,s), y_3(\alpha,\beta,s)] d\alpha d\beta ds / d^2 > 0$$

$$H_s = \int_V \frac{\partial p [y_1(\alpha,\beta,s), y_2(\alpha,\beta,s), y_3(\alpha,\beta,s)]}{d^2 \partial s} R_{cr} d\alpha d\beta ds \quad ,$$

where $R_k(y_1, y_2, y_3), p(y_1, y_2, y_3)$ are the defined values of the laminar solution and continuity equation; the function of external influence $h_l(y_1, y_2, y_3)$ is also defined. Micro-roughness, which is located along the pipeline, determines the critical Reynolds number. This micro roughness is less than macro roughness, which affects the resistance coefficient at the high Reynolds number. However, since the Reynolds number depends on the geometry of the pipeline through its diameter, the critical Reynolds number is only inversely proportional to the rms of the tangent of slope micro roughness and depends on the geometry of the pipeline.

The coordinates of equilibrium positions are determined from the quadratic equation

$$\alpha_s^2 - \alpha_s \frac{2R_{cr} G_s}{F_s} + \frac{H_s}{F_s} = \alpha_s^2 - 2R_{cr}^s \alpha_s + T_s \gamma_s = 0, T_s = \frac{\Delta P_s d^3 R_{cr}}{\rho^2 \nu^2 L}, R_{cr}^s = \frac{R_{cr} G_s}{F_s} .$$

It should be noted that the laminar solution is refined and has the form $\alpha_s = R_{cr}^s - \sqrt{(R_{cr}^s)^2 - T_s\gamma_s}$, which becomes the linear laminar solution $\alpha_s = T_s\gamma_s /(2R_{cr}^s)$ with small differential pressure.

However, the turbulent formula of accounting the roughness is fair due to the same method of averaging in the turbulent regime

$$\frac{\alpha_s}{\sqrt{T_s}} = \frac{R_{cr}^s}{\sqrt{T_s}} - i\sqrt[4]{\gamma_s - \frac{(R_{cr}^s)^2}{T_s}}\lambda = \sqrt{\frac{(R_{cr}^s)^2}{T_s} + \sqrt{\gamma_s - \frac{(R_{cr}^s)^2}{T_s}\lambda^2}}\exp(i\varphi),$$

$$\lambda = 1/[k(T_s,\xi_0)R_{cr}/1.4l(T_s,\xi_0)+1]^{0.365}$$

where $k(T_s,\xi_0)/l(T_s,\xi_0)$ is the effective rms of the tangent of slope roughness, ξ_0 is a ratio of height of roughness to the pipeline's radius, where the critical Reynolds number $\alpha_s = R_{cr}^s$ satisfies the value of the Reynolds number corresponding to the beginning of the comprehensive solution. At the same, we obtain a solution with a laminar low Reynolds number. But this is not the final step in the solution of the problem of obtaining the turbulent solution. It is necessary to determine the effect of surface roughness; therefore, the use of experimental data is necessary. In principle, it is necessary to have a precise value of depending on the Reynolds number's dependence on micro roughness for a smooth surface.

3. Calculation of the pipeline with a round cross section in the case of an incompressible fluid

Calculation of the pipeline with a round cross section is an implementation of our algorithm. We can find a solution of the problem of any pipeline with a circular cross section in $V_z = V_0(t)[1 - r^2/a^2(z)]$ in cylindrical coordinates. Since there is an external influence only for the longitudinal axis $P(z) = P_2 + \frac{P_1 - P_2}{L}z$, where P_2, P_1 are the pressure in the start and end of the pipeline, P_2, P_1 are the length of pipeline; we also we neglect radial and angular velocity. The external inflience is equal to $h_z = \frac{P_1 - P_2}{L}$. According to formula (1.2), the pressure gradient is $\frac{\partial P}{\partial z} = \frac{P_1 - P_2}{L}$.

We get the following equation:

$$\frac{\partial V_z}{\partial t} + V_z\frac{\partial V_z}{\partial z} = -\frac{P_1 - P_2}{L} + \nu\Delta V_z.$$

One substitutes the value of the Reynolds number, it is obtained

$$\frac{\partial V_0}{\partial t}(1 - r^2/a^2) + 2V_0^2(1 - r^2/a^2)\frac{r^2}{a^3}\frac{da}{dz} = -\frac{P_1 - P_2}{\rho L} - \nu\frac{4V_0}{a^2}.$$

One multiplies the equation radius and integrates its radially, because it is a cylindrical coordinate system

$$\frac{\partial V_0}{\partial t}a^2/6 + \frac{(P_1 - P_2)a^2}{2\rho L} + 2\nu V_0 = -V_0^2\frac{ada}{6dz}.$$

If we take the module of the right side and find the rms deviation, we obtain the following equation:

$$\frac{\partial V_0}{\partial t}a^2/6 + \frac{(P_1 - P_2)a^2}{2\rho L} + 2\nu V_0 = V_0^2\frac{a\langle|da/dz|\rangle}{6} = V_0^2\frac{2ak}{l}. \quad (2.1)$$

If you select the minus sign to the value of rms deviation, we find that the roughness increases the flow rate, because the total derivative $\frac{dV_0}{dt}$ increases. However, this is wrong, because the flow rate should be reduced due to the roughness. Introducing a turbulent viscosity, we use a negative sign to the rms of the tangent of slope roughness, which is related to the correlation function of speed of this process

$-\rho < u_l' \; u_k' >= \rho K \dfrac{\partial < u_l' >}{\partial x_k}$ (see Monin & Yaglom, 1965), which leads to the plus sign at the rms member. More than that, you should choose the plus sign at the rms of the tangent of slope roughness in order to obtain a comprehensive turbulent solution. Otherwise, the solution describing the turbulent pulsating mode will not turn.

Passing from the radius to the pipe diameter and dividing by $v^2 k/(dl)$, we obtain

$$\frac{dR_0}{d\tau} = R_0^2 - 2R_0 R_{cr} + \frac{T}{8}; T = \frac{(P_2 - P_1) d^3 R_{cr}}{\rho v^2 L} \quad . (2.2)$$

$$\tau = 24t \cdot v/(R_{cr} d^2), R_0 = V_0 d/v, 1/R_{cr} = <| da/dz |>/12 = k/l = <| \tan\alpha |>$$

If micro roughness is distributed throughout the surface pipeline, it is also on macro roughness and determines the critical Reynolds number and the resistance coefficient at the Reynolds number of 2300. Micro roughness has a molecular nature and is determined by the average size of nucleus, which is equal to the geometric mean of the number with a coefficient; it is $\sigma = \sqrt{0.7155 r_A a_0}$ between the size of nucleus and the size of an atom, with a distance between the atoms, it is $a = 2.87a_0$. The distance between iron atoms is $a_{Fe} = 2.87a_0$, it is $a_{Ti} = 3.46a_0$ between titanium atoms, and it is $a_C = 3.567a_0$ between carbon atoms (see Kikoin, 1976). Thus, the absolute value of the tangent of the height slope of roughness of the metal surface in the pipeline is determined by the following formula

$$h(z) = <| \tan\alpha |> = \sum_{n=-N}^{N} \exp[-(z-na)^2/2\sigma^2]/\sqrt{2\pi} .$$

The rms slope of the tangent is

$$\frac{2}{R_{cr}} = \int_{-\infty}^{\infty} h(z) \frac{dz}{4\pi Na} = \frac{\int_{-\infty}^{\infty} \exp[-(z-na)^2/2\sigma^2] dz}{2\pi a} = \frac{\sigma}{\sqrt{2\pi} a} = .$$

$$= \frac{1}{2.87} \sqrt{\frac{r_A}{2\pi 0{,}7155 \cdot a_0}} = \frac{1}{2.87} \sqrt{\frac{1.4 \cdot 10^{-13}}{2 \cdot 3.1416 \cdot 0.7155 \cdot 0.5 \cdot 10^{-8}}} = \frac{1}{1150}$$

This value determines the critical Reynolds number by using the radius instead of the diameter in the Reynolds number. When using the diameter, the critical Reynolds number is $R_{cr} = 2300$. It should be noted that step changes in height of the surface are related to micro roughness. Its tangent of the slope forms a delta function and contributes to the critical Reynolds number in averaging in a given section $\frac{2}{R_{cr}} = \int_{-\infty}^{\infty} \sum_{k=1}^{P} \frac{|\Delta a_k|}{2\pi Pa} \delta(z - z_k) dz = \sum_{k=1}^{P} \frac{|\Delta a_k|}{2\pi Pa}$, where the jump height occurs in different corners of this section.

The macro roughness $<| da/dz |>$ is rarer and determines the resistance coefficient in the Reynolds number with the value of 12 and higher.

We obtain the stationary condition for the Navier-Stokes equations, considering one member of a number of solutions in one-dimensional case

$$R_0^2 - 2R_0 R_{cr} + T/8 = 0 .$$

In one-dimensional case with a constant cross-section of the pipeline, the continuity equation is satisfied identically. The laminar solution of this equation is

$$R_0 = R_{cr} - \sqrt{R_{cr}^2 - T/8} = [R_{cr}/\sqrt{T} - \sqrt{R_{cr}^2/T - 1/8}]\sqrt{T} .$$

When the external pressure $T = 8R_{cr}^2$, the comprehensive solution and the turbulent regime start, because the Reynolds number at this point is equal to the critical value. From the experiment, we have the critical Reynolds number for a round pipeline $R_{cr}=\frac{l}{k}=\frac{1}{<|\tan\alpha|>}=2300$. The resistance coefficient of the pipeline with a circular cross-section is determined by the following formula (one inserts the pressure drop in the formula, expressed in terms of the dimensionless pressure):

$$\lambda=\frac{2\Delta P_L d}{\rho V_a^2 L}=\frac{2Tv^2 k}{V_a^2 d^2 l}=\frac{2T}{R_{cr}\,|R_a^2|},$$

The average speed entering the Reynolds number is

$$V_a=\int_0^a rV_0(1-\frac{r^2}{a^2})dr/\int_0^a rdr=V_0/2, R_a=\frac{V_a d}{\nu}=\frac{R_0}{2}.$$

Thus, the asymptotic behavior of the resistance coefficient of the pipeline with a circular cross section for the laminar flow regime λ_{lam} is calculated correctly.

$$R_a=R_0/2=(R_{cr}-\sqrt{R_{cr}^2-T/8})/2\cong\frac{T}{32R_{cr}},\frac{T}{8R_{cr}^2}<<1,\lambda_{lam}=\frac{2T}{R_{cr}\,|R_a^2|}=\frac{64}{|R_a|}.$$

The asymptotic behavior is obtained with a small Reynolds number, when the convective term is small.

In case of a large pressure drop, we get a complex turbulent solution $R_0=R_{cr}-i\sqrt{T/8-R_{cr}^2}=(R_{cr}/\sqrt{T}-i\sqrt{1/8-R_{cr}^2/T})\sqrt{T}$.

If we calculate more specifically, the contribution of the rotational imaginary part to the translational speed of the stream movement corresponds to the root of the imaginary part according to the formula (2.3).

$$\frac{R_0}{\sqrt{T}}=\frac{R_{cr}}{\sqrt{T}}-i\sqrt[4]{\frac{1}{8}-\frac{R_{cr}^2}{T}}\beta=\sqrt{\frac{R_{cr}^2}{T}+\sqrt{\frac{1}{8}-\frac{R_{cr}^2}{T}}\beta^2}\exp(i\varphi). \quad (2.3)$$

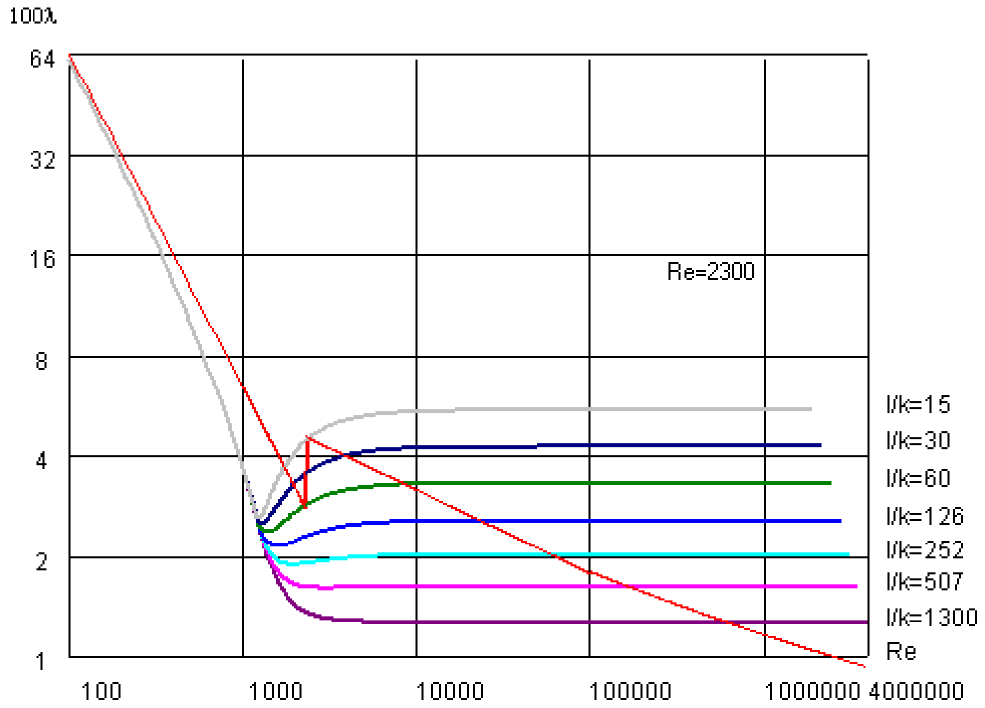
$$\beta=1/[k(T,\xi_0)R_{cr}/1.4l(T,\xi_0)+1]^{0.365}$$

Moreover, it is necessary to use the value of the ratio of the Reynolds number at the root of the dimensionless pressure as the value of member's order in the turbulent regime. It is worth mentioning, the diameter of the pipeline is defined up to $<d^2>=d^2/\{[k(T,\xi_0)R_{cr}/1.4l(T,\xi_0)+1]\}^{0.365}$. When macro roughness is at zero, the effective diameter is equal to the exact value, i.e. the effective diameter decreases with increasing the degree of surface roughness. Also, the ratio of the tangent of the slope of macro roughness towards micro roughness is more than $k/(l<|\tan\alpha|>)=1$. If we calculate the change in pipeline's diameter, the diameter decreases in $(1/1.4+1)^{0.1825}=1.1$ times (according to the empirical formula for reducing the mean square value of the diameter). With the ratio $l/k=30$, we get a reduction in the pipeline's diameter in $[2300/(30\cdot1.4)+1]^{0.1825}=2$ times.

Furthermore, the diameter varies only at the coefficient of a pulsating part of the solution, i.e. in the imaginary part. Modifier $\beta=1/[k(T,\xi_0)R_{cr}/1.4l(T,\xi_0)+1]^{0.365}$ appears from this part, because the imaginary part is proportional $\sqrt{T}\sim d^2$. The square root $\sqrt[4]{1/8-R_{cr}^2/T}$ corresponds to the average diameter.

It is necessary to consider the effect of wall roughness of the turbulent flow in the imaginary flow part of the Reynolds number. In order to receive the charts with a constant height of roughness, we introduce the effective rms of the tangent of slope. The effective rms of the tangent of slope roughness should depend on the external pressure $\frac{k(T,\xi_0)}{l(T,\xi_0)}$. At infinity of the Reynolds number or dimensionless pressure, we have the roughness that corresponds to a constant height of roughness $\frac{k(\infty,\xi_0)}{l(\infty,\xi_0)}=\frac{k}{r_0}=\frac{1}{\xi_0}$.

The formula is chosen in such a way that we can correctly determine the dependence of the Reynolds number in relation to the external pressure and to the resistance coefficient of the pipeline in infinity of the Reynolds number, as well as the external pressure $\operatorname{Im} R_0 = -i\sqrt[4]{1/8}/[k(\infty,\xi_0)R_{cr}/1.4l(\infty,\xi_0)+1]^{0.365}\sqrt{T}$ with the resistance coefficient that is equal to $\lambda = \frac{8}{R_{cr}\sqrt{1/8}/(R_{cr}/1.4\xi_0+1)^{0.73}}$. With the constant rms of the tangent of slope roughness $\frac{k}{l}$ and with different height of roughness k, we have a chart that is different from the classic Nikuradze's graphs.



Graph 1. Dependence of the resistance coefficient of the round pipeline on the Reynold number with different rms of the tangent of slope roughness.

Nevertheless, the Nikuradze formula is obtained with a constant ratio of the pipeline's radius r_0 to the average height of roughness k. The formula (2.3) provides an effective rms of the tangent of slope roughness, which is expressed in terms of the ratio of pipeline's radius to the average height of roughness through the dimensionless pressure.

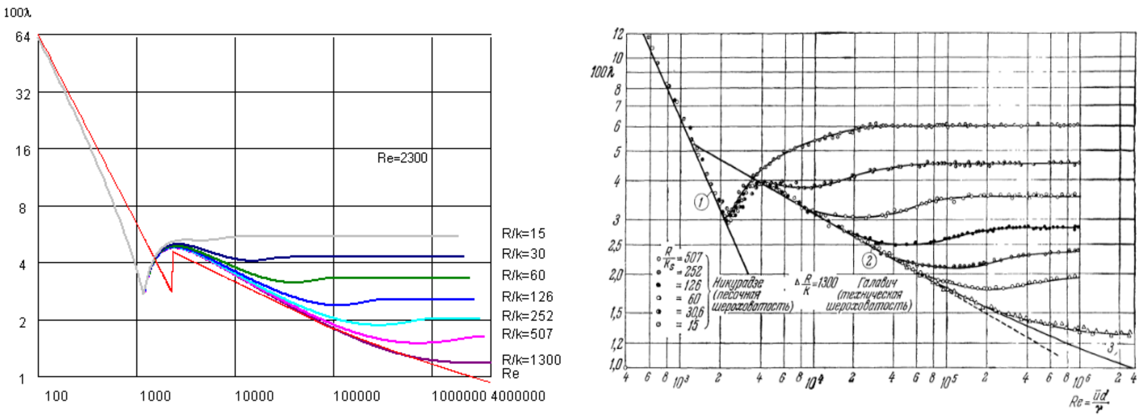
$$\frac{l(T,\xi_0)}{\delta(T,\xi_0)} = \{2\exp[-|\sqrt{T}-\sqrt{T_{cr}}|/|\alpha(\xi_0)|] + \xi_0[1-\exp(-|\sqrt{T}-\sqrt{T_{cr}}|/|\alpha(\xi_0)|)]\}\times$$
$$\times\{1+0.4\exp\{-[\sqrt{T}-\sqrt{T_{cr}}\beta(\xi_0)]/\gamma(\xi_0)\}\}, \xi_0 = r_0/k$$

$T_{cr} = 8R_{cr}^2$. Influence of an effective rms of the tangent of slope roughness on the flow properties depends on the Reynolds number or pressure differential. The empirical formula for determination of the coefficients $\alpha(\xi_0), \beta(\xi_0), \gamma(\xi_0)$ is

$$\alpha(\xi_0) = R_{cr}\frac{\xi_0}{1.5},\ \beta(\xi_0) = \frac{\xi_0}{5},\ \gamma(\xi_0) = 15R_{cr}(\frac{\xi_0}{30})^{1.5}.$$

It is worth mentioning that at the beginning of the formation of the imaginary part of a comprehensive solution $T = T_{cr}$ or at the beginning of turbulent solutions, the tangent of slope roughness is approximately equal to 2, and graphs are the same at different tangents of slope roughness.

The resistance coefficient of the flow in a circular pipeline is determined by the formula $\lambda = \frac{2T}{R_{cr} \, | R_a |^2}$, while the Reynolds number, which is calculated on the basis of the average flow rate is $R_a = R_0 / 2$. There is a graph solution that is obtained using a single number (see graph 2).



Graph 2. Theoretical and experimental dependence of resistance coefficient of the flow in a circular pipeline on the Reynolds number with different degrees of roughness.

In order to compare theoretical and experimental graph of dependence of the resistant coefficient of the flow resistance on the Reynolds number, we consider the graph of Nikuradze. Thus, the error of theoretical graph with respect to the experimental one is about 10%. However, the chart shown in the graph of Nikuradze is inaccurate in its laminar part of the solution, because the contrast ratio $\lambda = 64 / R$ comes as we approach the critical Reynolds number, which is reflected in the theoretical graph and little appears in the experimental one. The experimental graph for the logarithm of the pressure and the Reynolds number is shown by a straight line; however, there should be a distinction from a straight line with an increase in influence of the convective term.

This graph of the solution is obtained for the constant flow temperature; therefore, it will not change in case of the weak dependence of kinematic viscosity on the temperature of the formula. To implement it in case of turbulent flow, it is necessary to substitute the reduced pressure and the ratio of pipeline's radius to the height of roughness in the formula.

$$| R_0 | = \sqrt{R_{cr}^2 + \sqrt{T^2 / 8 - T R_{cr}^2 \beta^2}}$$

$$\beta = 1/[k(T,\xi_0) R_{cr} / 1.4 l(T,\xi_0) + 1]^{0.365}$$

$$\frac{l(T,\xi_0)}{k(T,\xi_0)} = \{2\exp[-|\sqrt{T} - \sqrt{T_{cr}}\,| / \alpha(\xi_0)] + \xi_0[1 - \exp(-|\sqrt{T} - \sqrt{T_{cr}}\,| / \alpha(\xi_0))]\} \times$$

$$\times \{1 + 0.4\exp\{-[\sqrt{T} - \sqrt{T_{cr}}\,\beta(\xi_0)] / \gamma(\xi_0)\}, \xi_0 = \frac{r_0}{\delta_0}$$

Moreover, the formula is constructed in such a way that this condition is satisfied $\frac{l(\infty,\xi_0)}{k(\infty,\xi_0)} = \xi_0$. In case of the laminar flow regime, there is a simple formula for determining the Reynolds number: $R_0 = R_{cr} - \sqrt{R_{cr}^2 - T/8}$.

4. Conclusion.

Based on the solution of Navier-Stokes equations, the graphs of dependence of the resistance coefficient of the round pipeline on the Reynold number with a different tangent of slope roughness and a constant high of roughness. In addition, we received the final formula for the dependence of the Reynolds number flow from the external pressure in laminar and turbulent regime, as well as the dependence of the flow resistance on the external pressure. However, the problem of roughness remains theoretically unresolved. This causes the need to bring experimental data in order to determine the effect of roughness. The characteristic size of the body and its surface roughness are common to different tasks; therefore, they should be considered equally for body movements in both a viscous compressible medium and in a circular pipeline with an incompressible fluid. The transition from the rms of the tangent of slope roughness to the constant height of roughness is the same for different tasks. However, there are other kinds of roughness such as a sinusoidal height of roughness. We should find an effective tangent of slope roughness for them. Moreover, a different type of roughness affects the imaginary part of the Reynolds number.

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